

General Certificate of Education (A-level) January 2013

Mathematics

MPC1

(Specification 6360)

Pure Core 1

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
−x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a) (i)	21 + 5k = 1			condone $3 \times 7 + 5k = 1$
	$\Rightarrow k = -4$	B1	1	AG condone $y = -4$
(ii)	(x =) 2 $(y =) -1$	B1 B1	2	midpoint coords are (2, -1)
(b)	$y = \frac{1}{5} - \frac{3}{5}x$	M1		obtaining $y = a \pm \frac{3}{5}x$
	(Gradient $AB = $) $-\frac{3}{5}$	A1	2	or $\frac{\Delta y}{\Delta x} = \frac{-4-2}{73}$ or $\frac{-1-2}{23}$ or $\frac{-41}{7-2}$ condone one sign error in expression allow -0.6 , $\frac{6}{-10}$ etc for A1 & condone error in rearranging if gradient is correct.
(c)	Perp grad = $\frac{5}{3}$	M1		-1/ "their" grad AB
	$y-2 = \frac{5}{3}(x+3)$ or $y = \frac{5}{3}x+c$, $c = 7$ etc	A1		correct equation in any form (must simplify $x3$ to $x+3$ or c to a single term equivalent to 7)
	5x - 3y + 21 = 0	A1	3	or any multiple of this with integer coefficients –terms in any order but all terms on one side of equation
(d)	3x + 5y = 1 and $5x + 8y = 4\Rightarrow P \ x = Q or R \ y = Sx = 12y = -7$	M1 A1 A1	3	must use correct pair of equations and attempt to eliminate y (or x) (generous) $(12, -7)$
	m		- 11	
	Total		11	

Q	Solution	Marks	Total	Comments
2(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) = \frac{4t^3}{8} - 2t$	M1 A1	2	one of these terms correct all correct (no $+ c$ etc)
(b)(i)	$t = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{4}{8} - 2$	M1		Correctly sub $t = 1$ into their $\frac{dy}{dt}$
	$=-1\frac{1}{2}$	A1cso	2	must have $\frac{dy}{dt}$ correct (watch for t^3 etc)
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}t} < 0$			must have used $\frac{dy}{dt}$ in part (b)(i)
	\Rightarrow (height is) decreasing (when $t = 1$)	E1√	1	must state that " $\frac{dy}{dt}$ < 0" or "-1.5 < 0" or the equivalent in words
				FT their value of $\frac{dy}{dt}$ with appropriate explanation and conclusion
(c)(i)	$\left(\frac{d^2y}{dt^2}\right) = \frac{4}{8} \times 3t^2 - 2$ $\left(t = 2, \frac{d^2y}{dt^2}\right) = 4$	M1		Correctly differentiating their $\frac{dy}{dt}$ even if wrongly simplified
	$\left(t=2, \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \right) 4$	A1cso	2	Both derivatives correct and simplified to 4
(ii)	\Rightarrow minimum	E1√	1	FT their numerical value of $\frac{d^2y}{dt^2}$ from part (c) (i)
	Total		8	

Q	Solution	Marks	Total	Comments
3(a)(i)	$\sqrt{18} = 3\sqrt{2}$	B1	1	Condone $k = 3$
(ii)	$\frac{2\sqrt{2}}{3\sqrt{2}+4\sqrt{2}}$	M1 A1		attempt to write each term in form $n\sqrt{2}$ with at least 2 terms correct correct unsimplified
	$=\frac{2}{7}$	A1	3	
				or $\times \frac{\sqrt{2}}{\sqrt{2}}$ M1 integer terms = $\frac{4}{6+8}$ A1 = $\frac{2}{7}$ A1
(b)	$\frac{7\sqrt{2} - \sqrt{3}}{2\sqrt{2} - \sqrt{3}} \times \frac{2\sqrt{2} + \sqrt{3}}{2\sqrt{2} + \sqrt{3}}$	M1		
	(numerator =) $14 \times 2 - 2\sqrt{6} + 7\sqrt{6} - 3$	m1		correct unsimplified but must simplify $\left(\sqrt{2}\right)^2$, $\left(\sqrt{3}\right)^2$ and $\sqrt{2} \times \sqrt{3}$ correctly
	(denominator = 8 - 3 =) 5	B1		must be seen or identified as denominator giving $\frac{25+5\sqrt{6}}{5}$
	$(Answer =) 5 + \sqrt{6}$	A1cso	4	m = 5, n = 6
	Total		8	

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Q	Solution	Marks	Total	Comments
4(a)(i)	$(x-3)^2$	M1		or $p = 3$ seen
	$(x-3)^2 + 2$	A1	2	
(ii)	$(x-3)^2 = -2$ No (real) square root of -2 therefore	M1		FT their positive value of <i>q</i> not use of discriminant for graphical approach see below to see if
	equation has no real solutions	Alcso	2	SC1 can be awarded
(b)(i)	x = 'their' p or $y =$ 'their' $qVertex is at (3, 2)$	M1 A1cao	2	or $x = 3$ found using calculus
(ii)	V	B1		y intercept = 11 stated or marked on y-axis (as y intercept of any graph)
	11	M1		
		A1	3	above x-axis, vertex in first quadrant crossing y-axis into second quadrant
(iii)	Translation	E1		and no other transformation
	through $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$	M1		FT negative of BOTH 'their' vertex coords
		A1	3	both components correct for A1; may describe in words or use a column vector
	Total		12	

Q Q	Solution	Marks	Total	Comments
5(a)	$p(-1) = (-1)^3 - 4 \times (-1)^2 - 3(-1) + 18$	M1		p(-1) attempted not long division
	(=-1-4+3+18) = 16	A1	2	
(b)(i)	$p(3) = 3^3 - 4 \times 3^2 - 3 \times 3 + 18$	M1		p(3) attempted not long division
	$p(3)=27-36-9+18=0 \implies x-3 \text{ is a factor}$	A1	2	shown = 0 plus statement
(ii)	Quadratic factor $(x^2 - x + c)$ or $(x^2 + bx - 6)$	M1		−x or −6 term by inspection
	Quadratic factor $(x^2 - x - 6)$	A1		or full long division by $x-3$ or comparing coefficients or p(-2) attempted correct quadratic factor (or $x+2$ shown to be factor by Factor Theorem)
	[p(x)=] (x-3)(x-3)(x+2)	A1	3	or $[p(x)=] (x-3)^2 (x+2)$ must see product of factors
(c)	y †	M1		cubic curve with one maximum and one minimum
	-2 3 x	A1		meeting x -axis at -2 and touching x -axis at 3
	Final A1 is dependent on previous A1 and can be withheld if curve has very poor curvature beyond $x = 3$, V shape at $x = 3$ etc	A1	3	graph as shown, going beyond $x = -2$ but condone max on or to right of y-axis
	Total		10	

Q	Solution	Marks	Total	Comments
6(a)	(Gradient = $10 - 6 + 5$) = 9	B1		correct gradient from sub $x = 1$ into $\frac{dy}{dx}$
	y-4 = "their 9" (x-1) or $y = "their 9" x+c$ and attempt to find c using $x = 1$ and $y = 4$	M1		must attempt to use given expression for $\frac{dy}{dx}$ and must be attempting tangent and not normal and coordinates must be correct
	y = 9x - 5	A1	3	condone $y = 9x + c, \dots c = -5$
(b)	$(y=)\frac{10}{5}x^5 - \frac{6}{3}x^3 + 5x + C$	M1		one term correct
		A1 A1		another term correct all integration correct including $+ C$
	$4 = 2 - 2 + 5 + C$ $\Rightarrow C = -1$	m1		substituting both $x = 1$ and $y = 4$ and attempting to find C
	$y = 2x^5 - 2x^3 + 5x - 1$	A1cso	5	must have $y =$ and coefficients simplified
	Total		8	

Q	Solution	Marks	Total	Comments
7(a)	$x = 0 \Rightarrow y^2 - 4y - 12 \ (=0)$	M1	20002	sub $x = 0$ & correct quadratic in y
				$or (y-2)^2 = 16 or (y-2)^2 - 16 = 0$
	(y-6)(y+2) (=0)	A1		correct factors
				or formula as far as $\frac{4 \pm \sqrt{64}}{2}$
				or $y - 2 = \pm \sqrt{16}$
	$\Rightarrow y = -2, 6$	A1	3	condone (0, -2) & (0, 6)
(b)	$(x+3)^2 - 9 + (y-2)^2 - 4 (=12)$	M1		correct sum of square terms and attempt to complete squares (or multiply out) PI by next line
	$(r^2 =) 9+4+12$	A1		$(r^2 =)25$ seen on RHS
	$(\Rightarrow r =) 5$	A1	3	$r = \sqrt{25}$ or $r = \pm 5$ scores A0
(c)(i)	$(CP^{2} =) (2-3)^{2} + (5-2)^{2}$ $\Rightarrow (CP =) \sqrt{34}$	M1		condone one sign slip within one bracket
	\Rightarrow $(CP =) \sqrt{34}$	A1	2	n = 34
(ii)	$PQ^{2} = CP^{2} - r^{2} = 34 - 25$ $(\Rightarrow PQ =) 3$	M1 A1	2	Pythagoras used correctly with values FT "their" <i>r</i> and <i>CP</i>
	(2)		-	
	Total		10	

Q	Solution	Marks	Total	Comments
8(a)	$2x^2 - x - 1 = 2kx - 3k$			equated and multiplied out
	$2x^{2} - x - 1 - 2kx + 3k = 0 OE$ $\Rightarrow 2x^{2} - (2k+1)x + 3k - 1 = 0$	B1	1	and all 5 terms on one side and = 0 AG (correct with no trailing = signs etc)
(b)(i)	$(2k+1)^2 - 4 \times 2(3k-1)$ $(2k+1)^2 - 4 \times 2(3k-1) > 0$	M1		clear attempt at $b^2 - 4ac$
	$(2k+1)^2 - 4 \times 2(3k-1) > 0$	B1		discriminant > 0 which must appear before the printed answer
	$4k^2 + 4k + 1 - 24k + 8 > 0$			
	$\Rightarrow 4k^2 - 20k + 9 > 0$	Alcso	3	AG (all working correct with no missing brackets etc)
(ii)	$4k^2 - 20k + 9 = (2k - 9)(2k - 1)$	M1		correct factors or correct use of
				formula as far as $\frac{20 \pm \sqrt{256}}{8}$
	critical values are $\frac{1}{2}$ and $\frac{9}{2}$	A1		condone $\frac{4}{8}$, $\frac{36}{8}$ etc here but must combine sums of fractions
	† \ /	M1		sketch or sign diagram including values
	$\frac{1}{2}$ $\frac{9}{2}$ k			+ - + 0.5 4.5
	$k < \frac{1}{2}, k > \frac{9}{2}$ Take their final line as their answer	A1	4	fractions must be simplified condone use of OR but not AND
	Total		8	
	TOTAL		75	